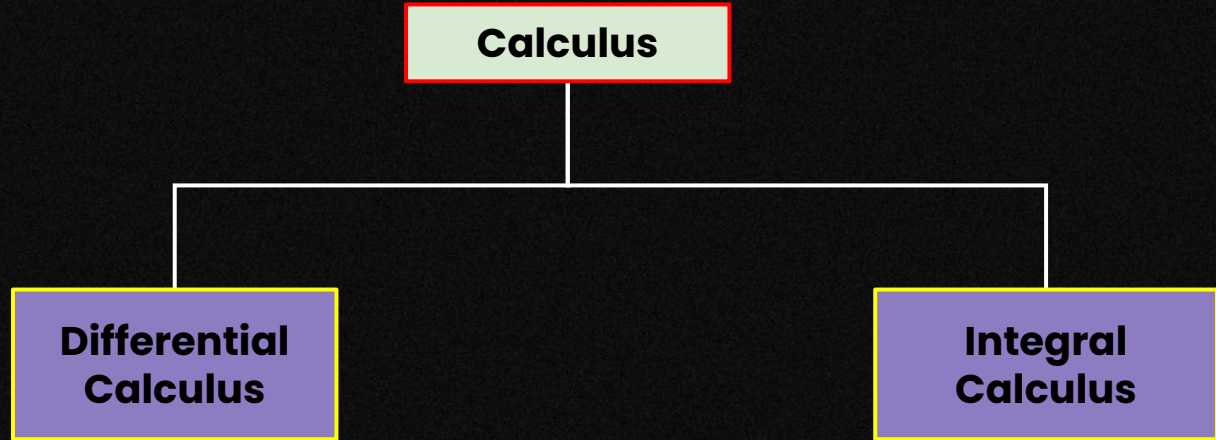


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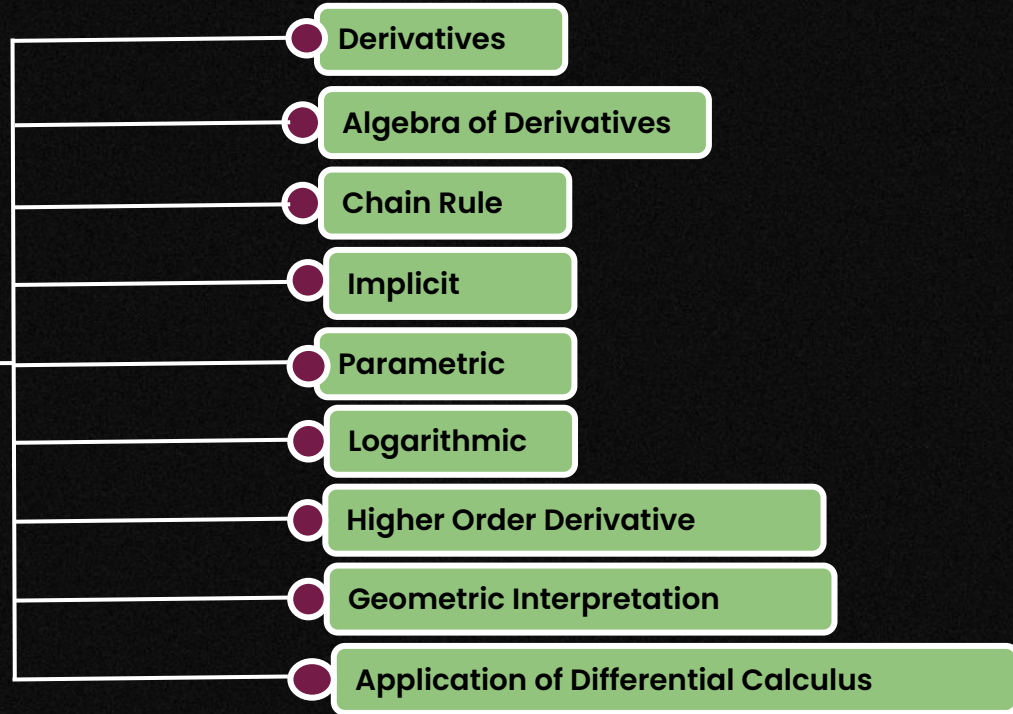
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Topics to be covered



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Derivative

- Let $y = f(x)$

Rate of change of y with respect to x is called derivative.

- The derivative of f is denoted by

- $f'(x)$

$$\frac{dy}{dx}$$

$$y'$$

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DERIVATIVE / DIFFERENTIAL COEFFICIENT



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- The derivative of $f(x)$ is also known as differential coefficient of $f(x)$ with respect to x .

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DIFFERENTIATION

- The process of finding derivative of a function is called differentiation.



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FORMULAE

$$(1) \frac{d}{dx} (x^n) = nx^{n-1}$$

$$(2) \frac{d}{dx} (e^x) = e^x$$

$$(3) \frac{d}{dx} (a^x) = a^x \log_e a$$

$$(4) \frac{d}{dx} (\text{constant}) = 0$$

$$(5) \frac{d}{dx} (e^{ax}) = ae^{ax}$$

$$(6) \frac{d}{dx} (\log x) = \frac{1}{x}$$



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ALGEBRA OF DERIVATIVES

$$(i) \frac{d}{dx}(u+v) = \left(\frac{du}{dx} + \frac{dv}{dx} \right)$$

$$(ii) \frac{d}{dx}(u-v) = \left(\frac{du}{dx} - \frac{dv}{dx} \right)$$

$$(iii) \frac{d}{dx}(uv) = \left(u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx} \right)$$

$$(iv) \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{\left(v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx} \right)}{v^2}$$



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ALGEBRA OF DERIVATIVE

$$\frac{d}{dx} \{c \cdot f(x)\} = c \cdot \frac{d}{dx} \{f(x)\}.$$

Where c be a real number .



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$$\frac{d}{dx} \{c \cdot f(x)\} = c \cdot \frac{d}{dx} \{f(x)\}.$$



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Example: Find the derivative of :

- 1) $8x^3$
- 2) $6\sqrt{x}$
- 3) $5e^x$

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$$\frac{d}{dx} \{f(x) + g(x)\} = \frac{d}{dx} \{f(x)\} + \frac{d}{dx} \{g(x)\}$$

Example: Differentiate with respect to x:

(a) $3x^2 + 5x - 2$



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PRODUCT RULE

$$\begin{aligned}\frac{d}{dx}[f(x)g(x)] &= f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)] \\ &= f(x)g'(x) + g(x)f'(x)\end{aligned}$$



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Example : Find the derivative of

$$xe^x$$



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$$\frac{d}{dx} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{g(x) \cdot \frac{d}{dx} \{f(x)\} - f(x) \cdot \frac{d}{dx} \{g(x)\}}{[g(x)]^2}$$

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Example: Differentiate

$$e^x / x$$



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CHAIN RULE

Example: Differentiate :

1) $(ax + b)^m$

2) $(3x + 5)^6$

3) $\sqrt{ax^2 + 2bx + c}$

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Example: Differentiate each of the following functions with respect to x :

(b) $a^x + x^a + a^a$



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Example: Differentiate each of the following functions with respect to x:

(c) $\frac{1}{3}x^3 - 5x^2 + 6x - 2\log x + 3$



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Example: Differentiate each of the following functions with respect to x :

(d) $e^x \log x$



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Example: Differentiate each of the following functions with respect to x:

(e) $2^x x^5$



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Example: Differentiate each of the following functions with respect to x:

$$(f) \frac{x^2}{e^x}$$



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Example: Differentiate each of the following functions with respect to x:

(g) $e^x / \log x$



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Example: Differentiate each of the following functions with respect to x:

(h) $2^x \cdot \log x$



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Example: Differentiate each of the following functions with respect to x:

(i) $\frac{2x}{3x^3+7}$



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Example: Differentiate each of the following functions with respect to x:

(h) $2^x \cdot \log x$



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Example: Differentiate $\log(1 + x^2)$ wrt. x



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IMPLICIT FUNCTIONS

- A function in the form $f(x, y) = 0$ is known as implicit function.
- **Example :**

$x^2y^2 + 3xy + y = 0$, where y cannot be directly defined as a function of x is called an implicit function of x .



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IMPLICIT FUNCTIONS



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- In case of implicit functions if y be a differentiable function of x , no attempt is required to express y as an explicit function of x for finding out $\frac{dy}{dx}$.
- In such case differentiation of both sides with respect of x and substitution of $\frac{dy}{dx} = y_1$ gives the result. Thereafter y_1 may be obtained by solving the resulting equation.

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Example: Find $\frac{dy}{dx}$ for $x^2y^2 + 3xy + y = 0$



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PARAMETRIC EQUATION

- When both the variables x and y are expressed in terms of a parameter (a third variable), the involved equations are called parametric equations.

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PARAMETRIC EQUATION

- For the parametric equations $x = f(t)$ and $y = h(t)$ the differential coefficient $\frac{dy}{dx}$ is obtained by using

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$



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Example: Find $\frac{dy}{dx}$ if $x = at^3$, $y = a / t^3$



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LOGARITHMIC DIFFERENTIATION

Product of function [$f(x) g(x)$]

Quotient of functions $\left[\frac{f(x)}{g(x)} \right]$



Function raised to the power
function $\{f(x)\}^{g(x)}$

- Then, it's derivative can be obtained easily by taking logarithm (with base e) on both sides and simplifying before differentiation. This procedure is called Logarithmic differentiation.

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LOGARITHMIC DIFFERENTIATION

$$y = [u(x)]^{v(x)}$$

- By taking logarithm (to base e) both the side above may be rewritten as

$$\log y = v(x) \log [u(x)]$$

- Using chain rule we may differentiate this to get

$$\frac{1}{y} \cdot \frac{dy}{dx} = v(x) \cdot \frac{1}{u(x)} \cdot u'(x) + v'(x) \cdot \log [u(x)]$$

- which implies that

$$\frac{dy}{dx} = y \left[\frac{v(x)}{u(x)} \cdot u'(x) + v'(x) \cdot \log [u(x)] \right]$$



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Example: Differentiate x^x w.r.t. x



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Some More Examples

Example: If $x^m y^n = (x+y)^{m+n}$ prove that $\frac{dy}{dx} = \frac{y}{x}$



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Some More Examples

Example: If $x^y = e^{x-y}$ prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$



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HIGHER ORDER DIFFERENTIATION



$$\text{Let } y = f(x) = x^4 + 5x^3 + 2x^2 + 9$$

$$\frac{dy}{dx} = \frac{d}{dx} f(x) = 4x^3 + 15x^2 + 4x = f'(x)$$

Since $f(x)$ is a function of x it can be differentiated again.

$$\text{Thus } \frac{d}{dx} \left(\frac{dy}{dx} \right) = f''(x) = \frac{d}{dx} (4x^3 + 15x^2 + 4x) = 12x^2 + 30x + 4$$

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HIGHER ORDER DIFFERENTIATION



$\frac{d}{dx} \left(\frac{dy}{dx} \right)$ is written as $\frac{d^2y}{dx^2}$ (read as d square y by dx square) and is called the second derivative of y with respect to x while $\frac{dy}{dx}$ is called the first derivative. Again the second derivative here being a function of x can be differentiated again and $\frac{d}{dx} \left(\frac{d^2y}{dx^2} \right) = f'''(x) = 24x + 30$.

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Example:

If $y = ae^{mx} + be^{-mx}$ prove that $\frac{d^2y}{dx^2} = m^2y$.



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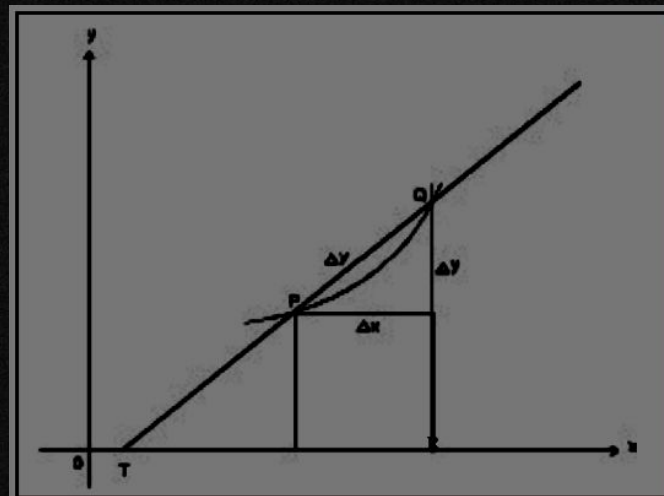
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GEOMETRIC INTERPRETATION OF THE DERIVATIVE



The derivative of $f(x)$ at a point x represents the slope (or sometime called the gradient of the curve) of the tangent to the curve $y = f(x)$ at the point x .



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Example: Find the gradient of the curve $y = 3x^2 - 5x + 4$ at the point $(1, 2)$.



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EXERCISE 8 (A)

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Choose the most appropriate option (a) (b) (c) or (d).

Que.1 The gradient of the curve $y = 2x^3 - 3x^2 - 12x + 8$ at $x = 0$ is

- (a) -12
- (b) 12
- (c) 0
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.2 The gradient of the curve $y = 2x^3 - 5x^2 - 3x$ at $x = 0$ is

- (a) 3
- (b) -3
- (c) $1/3$
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.3 The derivative of $y = \sqrt{x+1}$ is

- (a) $1/\sqrt{x+1}$
- (b) $-1/\sqrt{x+1}$
- (c) $1/2\sqrt{x+1}$
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.4 If $f(x) = e^{ax^2+bx+c}$ the $f'(x)$ is

- (a) e^{ax^2+bx+c}
- (b) $e^{ax^2+bx+c} (2ax + b)$
- (c) $2ax + b$
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.5 If $f(x) = \frac{x^2+1}{x^2-1}$ then $f'(x)$ is

(a) $-4x / (x^2 - 1)^2$

(b) $4x / (x^2 - 1)^2$

(c) $x / (x^2 - 1)^2$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.6 If $y = x(x - 1)(x - 2)$ then dy/dx is

(a) $3x^2 - 6x + 2$

(b) $-6x + 2$

(c) $3x^2 + 2$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.7 If $xy = 1$ then $y + dy/dx$ is equal to

- (a) 1
- (b) 0
- (c) -1
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.8 The derivative of the function $\sqrt{x+\sqrt{x}}$ is

(a) $\frac{1}{2\sqrt{x+\sqrt{x}}}$

(b) $1 + \frac{1}{2\sqrt{x}}$

(c) $\frac{1}{2\sqrt{x+\sqrt{x}}} \left(1 + \frac{1}{2\sqrt{x}}\right)$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.9 Given $e^{-xy} - 4xy = 0$, dy/dx can be proved to be

(a) $-y/x$

(b) y/x

(c) x/y

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.10 If $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$, dy/dx can be proved to be

(a) x/y

(b) $\frac{x}{\sqrt{x^2 - a^2}}$

(c) $\frac{1}{\sqrt{\frac{x^2}{a^2} - 1}}$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.11 If $\log (x / y) = x + y$, dy/dx may be found to be

(a) $\frac{y(1-x)}{x(1+y)}$

(b) y/x

(c) $1 - x/1 + y$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.12 If $f(x, y) = x^3 + y^3 - 3axy = 0$, dy/dx can be found out as

(a) $\frac{ay-x^2}{y^2+ax}$

(b) $\frac{ay-x^2}{y^2-ax}$

(c) $\frac{ay+x^2}{y^2+ax}$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.13 Given $x = at^2$, $y = 2at$; dy/dx is calculated as

- (a) t
- (b) $-1/t$
- (c) $1/t$
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.14 Given $x = 2t + 5$, $y = t^2 - 2$; dy/dx is calculated as

- (a) t
- (b) $-1/t$
- (c) $1/t$
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.15 If $y = \frac{1}{\sqrt{x}}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2x\sqrt{x}}$

(b) $\frac{-1}{x\sqrt{x}}$

(c) $-\frac{1}{2x\sqrt{x}}$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.16 If $x = 3t^2 - 1$, $y = t^3 - t$, then dy/dx is equal to

(a) $\frac{3t^2 - 1}{6t}$

(b) $3t^2 - 1$

(c) $\frac{3t - 1}{6t}$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.17 For the curve $x^2 + y^2 + 2gx + 2hy = 0$, the value of dy/dx at $(0, 0)$ is

- (a) $-g/h$
- (b) g/h
- (c) h/g
- (d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.18 If $y = \frac{e^{3x} - e^{2x}}{e^{3x} + e^{2x}}$, then $\frac{dy}{dx}$ is equal to

(a) $2e^{5x}$

(b) $1/(e^{5x} + e^{2x})^2$

(c) $e^{5x}/(e^{5x} + e^{2x})$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.19 Given $x = t + t^{-1}$ and $y = t - t^{-1}$ the value of dy/dx at $t = 2$ is

(a) $3/5$

(b) $-3/5$

(c) $5/3$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.20 If $x^3 - 2x^2y^2 + 5x + y - 5 = 0$ then dy/dx at $x = 1, y = 1$ is equal to

(a) $4/3$

(b) $-4/3$

(c) $3/4$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.21 The derivative of $x^2 \log x$ is

(a) $1 + 2 \log x$

(b) $x(1 + 2 \log x)$

(c) $2 \log x$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.22 The derivative of $\frac{3-5x}{3+5x}$ is

(a) $30/(3 + 5x)^2$

(b) $1/(3 + 5x)^2$

(c) $-30/(3 + 5x)^2$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.23 Let $y = \sqrt{2x} + 3^{2x}$ then $\frac{dy}{dx}$ is equal to

(a) $(1/\sqrt{2x}) + 2 \cdot 3^{2x} \log_e 3$

(b) $1/\sqrt{2x}$

(c) $2 \cdot 3^{2x} \log_e 3$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.24 The derivative of e^{3x^2-6x+2} is

(a) $30(1 - 5x)^5$

(b) $(1 - 5x)^5$

(c) $6(x - 1)e^{3x^2-6x+2}$

(d) None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.25 If $y = \frac{e^x + 1}{e^x - 1}$ then dy/dx is equal to

(a) $\frac{-2e^x}{(e^x - 1)^2}$

(b) $\frac{2e^x}{(e^x - 1)^2}$

(c) $\frac{-2}{(e^x - 1)^2}$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.26 If $x = at^2$, $y = 2at$ then $\left[\frac{dy}{dx} \right]_{t=2}$ is equal to

- a. $1/2$
- b. -2
- c. $-1/2$
- d. None of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.27 Let $f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2$ then $f'(2)$ is equal to

(a) $3/4$

(b) $1/2$

(c) 0

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.28 If $f(x) = x^2 - 6x + 8$ then $f'(5) - f'(8)$ is equal to

- (a) $f'(2)$
- (b) $3f'(2)$
- (c) $2f'(2)$
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.29 If $f(x) = x^k$ and $f'(1) = 10$ the value of k is

(a) 10

(b) -10

(c) 1/10

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.30 If $y = e^x + e^{-x}$ then $dy/dx - \sqrt{y^2 - 4}$ is equal to

- (a) 1
- (b) -1
- (c) 0
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.31 The derivative of $(x^2 - 1)/x$ is

(a) $1 + 1/x^2$

(b) $1 - 1/x^2$

(c) $1/x^2$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.32 The differential coefficients of $(x^2 + 1)/x$ is

(a) $1 + 1/x^2$

(b) $1 - 1/x^2$

(c) $1/x^2$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.33 If $y = e^{\sqrt{2x}}$ then dy/dx is equal to _____.

(a) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

(b) $e^{\sqrt{2x}}$

(c) $\frac{e^{\sqrt{2x}}}{\sqrt{2x}}$

(d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.34 If $x = (1 - t^2)/(1 + t^2)$ $y = 2t/(1 + t^2)$ then dy/dx at $t = 1$ is

- (a) $1/2$
- (b) 1
- (c) 0
- (d) none of these



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Choose the most appropriate option (a) (b) (c) or (d).

Que.35 $f(x) = x^2/e^x$ then $f'(1)$ is equal to _____.

- (a) $-1/e$
- (b) $1/e$
- (c) e
- (d) none of these



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